Gas Cycles

Turbojet Cycle

diffuser, combustion, nozzle

Produces thrust (vs. work)

Assumptions :

1 → 2 : Diffuser decelerates gas → ≈ 0 ms⁻¹

5 → 6 : $v_5 \approx 0\,\text{ms}^{-1}$, nozzle accelerates until $\underbrace{p_6 = p_1}$ inlet pressure

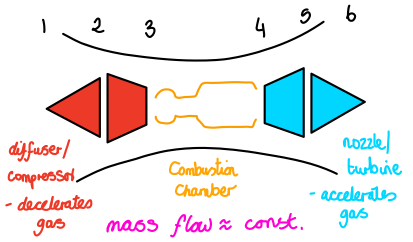
Turbine produces sufficient work to drive compressor, NO MORE

$$\text{Thrust} = \frac{d(mv)}{dt}$$

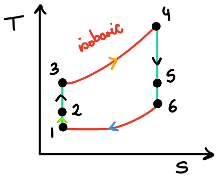
$$\boxed{F = \dot{m}V_6 - \dot{m}V_1}$$

leaving system bound. ($\dot{m}V_6$)

entering system bound. ($\dot{m}V_1$)



mass flow ≈ const.



combustion pressure

ambient pressure (e.g 0.6atm)

isentropic

Deceleration 1→2 : increase in temp as K.E transferred, and pressure increase

$p \propto T^{\frac{7}{2}}$ ∴ $T \propto p^{\frac{2}{7}}$

$m c_p \Delta T = \dot{Q}$ or $\dot{W}$ or $\dot{m}\left(\frac{\Delta v^2}{2}\right)$

Brayton (Joule) Cycle

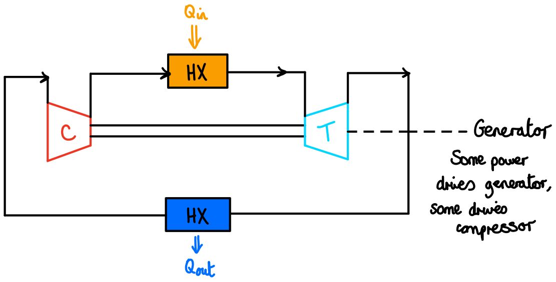
air compressed in seperate air compressor

fuel injection plus combustion

exhaust gases drive turbine

Assumptions :

- heat input replaces combustion
- heat output closes cycle

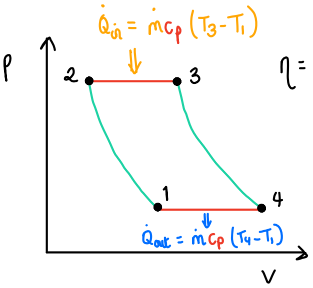


Some power drives generator, some drives compressor

Pressure ratio across compressor = pressure ratio across turbine

isobaric

isentropic



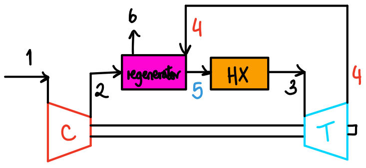
$\dot{Q}_{in} = \dot{m}c_p(T_3 - T_1)$

$\dot{Q}_{out} = \dot{m}c_p(T_4 - T_1)$

$$\eta = 1 - \frac{Q_{out}}{Q_{in}} = 1 - \frac{T_4 - T_1}{T_3 - T_2}$$

$$\boxed{\eta = 1 - \frac{1}{r_p^{\frac{\gamma-1}{\gamma}}}}$$

$$\text{Work Ratio} = \frac{\text{net work}}{\text{turbine work}}$$



5 at compressor exit

4 at turbine exit

To 'remove' irreversibility enthalpy must be recovered from turbine exhaust

$T_4 \geq T_5$, $T_4 = T_5$ best

$$e = \frac{\text{heat transfer}}{\text{max heat transfer}} = \frac{T_5 - T_2}{T_4 - T_2}$$

Air Standard Cycles

Assumptions

Heat rejection approximates exhaust

Heat input approximates combustion

Constant heat capacities

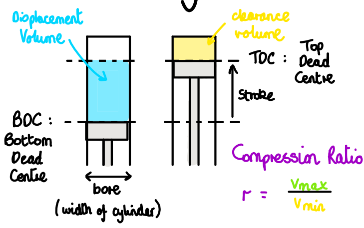
Gas treated as ideal gas with air properties

Gas circulates in closed loop

→ sealed vessel

→ exhaust gas cooled and recycled

Piston Cylinder



Compression Ratio :

$$r = \frac{V_{max}}{V_{min}}$$

↑r = ↑efficiency

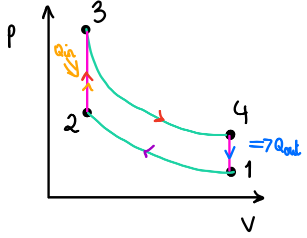
↳ however large r results in knocking too

V_{min} = clearance volume

V_{max} = clearance + displacement

Otto Cycle
(4 stroke, spark ignition)

2 isentropic, 2 isochoric



Adiabatic ∴ ΔQ = 0

1 → 2 : Compression Stroke → 1 stroke models 1 process

energy transferred out of flywheel

isentropic compression $\frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{\gamma-1}$

Power Stroke : 2→3 & 3→4 → 1 stroke models 2 processes

energy transferred into flywheel

2 → 3 : Ignition

Isochoric heating

↳ no work done → $Q_{in} = U = mc_v(T_3 - T_2)$

3 → 4 : Piston moves TOC → BOC

isentropic expansion ∴ $T_3 = T_4 r^{\gamma-1}$

→ $T_4 < T_3$

4 → 1 : Exhaust

Isochoric cooling → $Q_{out} = mc_v(T_4 - T_1)$

2 strokes, 1 process { Combustion gas expelled (BOC → TOC)
Air induction (TOC → BOC)

$$\frac{W}{Q_{in}} = \frac{Q_{in} - Q_{out}}{Q_{in}}$$

$$\therefore \eta = \boxed{1 - \frac{Q_{out}}{Q_{in}} = 1 - \frac{(T_4 - T_1)}{(T_3 - T_2)} = 1 - \frac{1}{r^{\gamma-1}}}$$

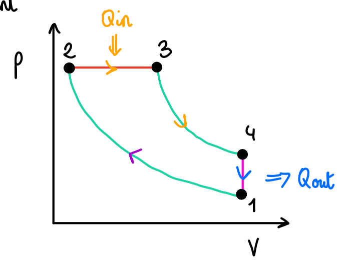
→ efficiency depends only on compression ratio

Diesel Cycle

- No spark plug, fuel injected at high pressure at TOC, intake pure air

∴ can operate at higher compressor ratios due to pure air ∴ more efficient

2 isentropic processes, 1 isobaric, 1 isochoric



1. Compression 1 → 2

$$r = \frac{V_1}{V_2}$$

Cut-off ratio

$$r_c = \frac{V_3}{V_2} = \frac{T_3}{T_2} \quad (\text{const. } p)$$

2. Ignition : 2 → 3, 3 → 4

Isobaric heating $\Delta Q = mc_p(T_3 - T_2)$

Isentropic expansion

3 & 4. Exhaust and Intake : 4 → 1

Isochoric cooling $\Delta Q = mc_v(T_4 - T_1)$

$$\eta = 1 - \frac{Q_{out}}{Q_{in}} = \frac{mc_v(T_4 - T_1)}{mc_p(T_3 - T_2)}$$

where $\frac{c_p}{c_v} = \gamma$

and $\boxed{T_2 = T_1 r^{\gamma-1}}$

$$\boxed{\eta = 1 - \frac{1}{\gamma r^{\gamma-1}}\left(\frac{r_c^{\gamma} - 1}{r_c - 1}\right)}$$